International Journal of Women in Technical Education and Employment
ISSN: 2811-1567. Volume 6 – Issue 1. July 2025
https://fpiwitedjournal.federalpolyilaro.edu.ng

Analysis of Conditional Distributions in Asymmetric MSGARCH Models for Volatility Forecasting: A Focus on Crude Oil Prices

Idowu, A. O.[1] & Adesanya, C. O.[2]
[1]Department of Mathematics and Statistics, Federal Polytechnic Ilaro, Ogun State, Nigeria.
[2]Department of Statistics, Yaba College of Technology Yaba, Lagos State, Nigeria.
adam.idowu@federalpolyilaro.edu.ng

ARTICLE HISTORY
Received: Maech 24, 2025
Revised: April 12, 2025
Accepted: April 18, 2025

Abstract
Volatility is an aspect of financial time series and a key factor in decision-making. This research uses the Markov Switching Generalized Autoregressive Conditional Heteroscedasticity (MS-GARCH) approach. The study aims to monitor the volatility conditions using asymmetric single regime switching dynamics, which deal with the non-linearity and heteroscedastic nature of the fluctuation in the price of crude oil. This research also explores the effects of conditional heteroscedasticity in evaluating crude oil price volatility. To analyze crude oil price, several distributions were compared based on maximum likelihood function used in estimating the model, two single regimes that monitor the volatility in the data on crude oil price, the GARCH model, generalized Error Distribution (GED), the student-t, and the Gaussian distribution, which were also used in this research. It is affirmed that, out of all the distributions compared, only GED exhibits the best fit in the analysis of the crude oil data set. The results show that among the compared distributions, Generalized Error Distribution exhibits the best fit, with a log-likelihood value of -1542.37, which performs better than Student-t (-1559.22) and Gaussian (-1574.89), respectively. Furthermore, the estimated GARCH parameters in the distribution signify volatility clustering and regime switching effects, enumerating the effectiveness of the model in capturing constant shifts in the prices of crude oil due to market volatility. The findings also reveal that the robustness of MS-GARCH approach in the price of crude oil provides an important insight for policymakers, financial analysts, and energy economists.

Keywords: Crude Oil Price, Financial Time Series, Generalized Error Distribution, Markov Switching GARCH Model, Maximum Likelihood Estimation

Citation
Idowu, A. O. & Adesanya, C. O. (2025). Analysis of Conditional Distributions in Asymmetric MSGARCH Models for Volatility Forecasting: A Focus on Crude Oil Prices. *International Journal of Women in Technical Education and Employment,* 6(1), 57 – 69.

Introduction
Asymmetric Markov Switching Generalized Autoregressive Conditional Heteroscedasticity (MS-GARCH) is one of the essential models in financial econometrics due to its ability to monitor the dependency in time series analysis. Bauwas et al. (2018) conducted their research on financial time series data and centered their work on the indispensable framework by modeling volatility dynamics. The results of their findings explain the variation in the impacts of stock market volatility by evaluating the leverage effects in the financial data set.

Ardia et al. (2019) also use the model parameter on the variance equation, thereby capturing its effectiveness across different groups of regimes. This flexibility gives adequate structure to financial time series, especially when there are unexpected changes in the market prices. Nguyen (2020) estimated the model by accounting for changes in the impacts of positive and Negative shocks on volatility, which is used in financial data sets.

Dias and Lopes (2021) In their research, they use different distributions as parameters, such as normal, student-t, and skewed distribution, to examine the model's ability, thereby combining heavy tail and symmetric properties of financial revert using the real conditional distribution. From all these findings, it was deduced that the model enhances forecasting accuracy because it involves an econometric data set.

Zivot (2022) MS-GARCH is also a major tool in financial econometrics, capturing non-linear dependent volatility in time series and analyzing market behavior during turbulence.

The study conducted by Nguyen et al. (2022) expounds on the importance of using appropriate conditional distributions, such as Laplace transformation, to enhance the model's ability. Hansen and Lunde (2023) compare several conditional distributions within MS-GARCH. It examines its performance under a different market system. Such comparisons are very useful in predicting risk management to understand the extreme prices in the sales of crude oil in the market and adequately forecasting its price volatility.

According to Ardia & Bluteau (2021), Bayesian methods are used to estimate and compute algorithms. The Reliability and accuracy of parameter estimates compared with Markov Chain Monte Carlo (MCMC) simulations. Zhang and Lee (2024). These advancements are driving the analysis of MSGARCH models using real-world situations, which include stress testing and portfolio optimization. Yamada and Watanabe (2020) formulate a volatility model in time-series analysis by ensuring a structural shift in the price of crude oil. The model is used to evaluate the dynamic regime shift, thereby reflecting on the behavior of the financial market, which includes the time of instability in price due to macroeconomic development and market crises.

Zhu et al. (2021) explain how the effectiveness of skewed and fat-tailed distributions captures the dynamic nature of the model in terms of returns. It was deduced that MS-GARCH is a model and a reliable method for evaluating a highly distributed array of financial time series data, which also signifies the model complexity in variables.

Park and Lee (2022) use Generalized Error Distribution (GED) to capture moderate volatility by comparing it to other distributions. In their findings, they noticed that skewed student t-distribution performs much better than symmetric pairs in capturing the tail behavior, most especially in times of market turbulence, which leads to pricing and risk management. Li et al. (2024) generalized error distribution (GED) captures moderate volatility since skewed t-distributions frequently perform better than symmetric pairs in capturing tail behavior, especially during market turbulence, such as options in pricing and risk management, providing great insights. Moreover, speeding up financial markets, such as cryptocurrencies and ESG investing, has enabled the use of asymmetric MSGARCH models.

He *et al*. (2023) study the models with flexible conditional distributions without asymmetric patterns. Since the ongoing development of computational techniques like efficient Bayesian algorithms and machine learning integrations, researchers have more accurate optimization in selecting and executing these distributions Wang & Zhang (2024). According to Jiang et al. (2023) the models provide easy compatibility and accuracy in capturing real-life events by imploring asymmetric conditional distributions, such as generalized t-distributions or skewed normal distributions. Liang et al. (2022). Conducted his research on varied capacities for handling skewness, kurtosis, and heavy tails distribution in financial data of the model; the mode of conditional distribution has a direct impact on model performance. Chowdhury et al. (2023) studied the conditional distributions within asymmetric MSGARCH frameworks, enumerating their importance in modeling the model to specific data characteristics. These models are crucial for portfolio optimization, risk management, and derivative pricing, where accurate volatility modeling directly influences the opinion.

Chen & Zhao (2024) use the MS-GARCH model in novel financial settings since the level of volatility is high due to its various regimes. Moreover, comparing the model has influenced the development in computing, such as Bayesian estimate techniques, which allows suitable configurations on various market price stability. The model also serves as a key tool in financial econometrics by accounting for two-regime-dependent volatility regimes.

Hansen et al. (2021) evaluate the conditional distribution from the backward shift of MS-GARCH, which determines whether the model best fits the observed data. The model also shows accuracy and prediction power. Kim & Lee (2022) centered their research on different market characteristics due to skewness in the distribution from the heavy tails, which is suitable for a hyperbolic generalized family of distribution. Sun et al. (2023) carried out their study on the conditional distribution, which captures the dynamic nature of the financial data set and results in high prediction accuracy. Liu et al. (2024) also use computational methods and apply them to the model using the Bayesian approach with the help of advanced sampling methods such as Markov Chain Monte Carlo (MCMC), allowing more precise estimation complexity conditional distribution. Machine learning algorithms were also used to model the adaptive mechanism, which is used in selecting optimal distribution on dynamic market conditions, leading to advancing and expanding the scope of the model, which is more volatile and efficient in modeling financial market complexities.

Bentes (2020) employed the model to analyze the leverage effect in volatility clustering. Yao and Wang (2021) The distribution is also important in the aspects of statistical properties of inventing which model is the best fit in financial time series data. Student t-distribution, Laplace, and skewness exhibit real-life market performance behavior, which enhances their influence in forecasting and risk management of the data.

A study by Nguyen et al. (2022) explained that the generalized hyperbolic distribution executes concrete alternatives in monitoring heavy-tailed distributional behavior, in which the asymmetric t-distribution generally adopts the modeling in skewness series. Chen et al. (2023) show that the method of conditional distributions affects the ability of the models to predict prices of Crude oil in the markets since the movements and accessibility to risk effectively Similarly, applications of the model in finance estimate the use of asymmetric MSGARCH models in interpreting the volatility patterns in ESG portfolios. Luo & Zhang (2023) use MSGARCH models in the development of computational econometrics. In complement with methods like integrated nested Laplace approximations (INLA) and Hamiltonian Monte Carlo (HMC), Bayesian inference has had a significant effect, which makes researchers evaluate complicated conditional distributions with accuracy. Furthermore, the selection of dynamic distribution has been imbibed through the integration of machine learning algorithms, enabling adaptive modeling of effecting market circumstances Wang & Li (2024). These advancements have greatly enhanced MSGARCH models in terms of accuracy and adaptability in reflecting the dynamics of financial market stability.

Therefore, this study aims to analyze and forecast the volatility in the price of crude oil using an Asymmetric Markov Switching GARCH (MS-GARCH) model, focusing on conditional distributions in capturing regime-dependent and asymmetric shocks. Comparing various performances of the distributions, such as the Generalized Error Distribution (GED), Student-t, and Gaussian, this study also identifies a suitable distribution for modeling real crude oil price dynamics. The study also examines how regime-switching dynamics influence model performance on structural changes and market turbulence. The goal is to provide a robust volatility forecasting model that helps in making decisions for policymakers, investors, and stakeholders in the energy sector.

Materials and Methods

The Price of Crude oil, which has a developing financial market, is a high-volatility market. Regimes considered to exist for volatility were handled in two categories: single regimes and two regimes, and both regimes were used to carry out the analysis.

Sources of Data

The data used for this analysis were obtained from the archives of the Central Bank of Nigeria (CBN). The data is monthly data of the Nigerian average Crude Oil Price. The data is from January 1990 to September 2021.

Methods of data analysis

The analytical-qualitative data technique adopted for research work was analyzed using descriptive and inferential statistics, while the qualitative data was analyzed using both the conditional, Gaussian, Student-t, and Generalized Error distributions for comparison of the Markov Switching Garch Model.

Markov Switching GARCH Models

The essential feature of regime-switching models is the possibility of some or all of the model's parameters switching for multiple regimes (or states of the world) according to a Markov process driven by a state variable, S_t Markov-switching. GARCH models (MSGARCH) have garnered much interest in recent years from researchers because they explain the high persistence of volatility found with single-regime GARCH models (Ardia, 2017).

The conditional variation equation for the GARCH (1,1) model in a regime-switching framework.

$$h_{it} = a_{0i} + a_{1i}x_{t-1}^2 + a_{2i}h_{t-1} \qquad (1)$$

where

h_{it} = is the conditional variance at period t in regime i = (1, 2)

h_{t-1} = is a state-independent average of past conditional variances

Gaussian Distribution

The normal (or Gaussian) distribution was first described by Carl Friedrich Gauss in 1809 in the context of measurement errors in astronomy. This distribution was applied extensively in the area of Applied Probability and Statistics.

Given a normal distribution.

$$f(xn, \mu, \sigma^2) = \left(\frac{1}{\sqrt{2\pi\sigma^2}} . \exp^{-\frac{1}{2}\left(\frac{(x_n-\mu)^2}{\sigma^2}\right)}\right) \qquad (2)$$

MLE of the Gaussian Distribution

The MLE of the Gaussian process, such as the likelihood function (L), is given by

$$L = p(X|\theta) = N(X|\theta) = N(X|\mu, \textstyle\sum) \qquad (3)$$

Therefore, the model for estimating both parameters, μ and Σ, is

$$\mu_{MLE} = argmax_{\mu} N(X|\mu, \textstyle\sum) \qquad (4)$$

$$\textstyle\sum_{MLE} = argmax_{\sum} N(X|\mu, \sum) \qquad (5)$$

by evaluating a normal Gaussian distribution.

$$\theta_{MLE} = argmax_{\theta} \log N(X|\theta) \qquad (6)$$

Assuming the model has a diagonal covariance matrix.

$$\sum_{n=1}^{N} \log (N(X_n|\mu, \textstyle\sum)) = \sum_{n=1}^{N} \log (N(X_n|\mu, \sigma^2)) \qquad (7)$$

Thus, we can have the fully specified log-likelihood function

$$\sum_{n=1}^{N} \log \left(N(X_n|\mu, \sigma^2)\right) = \sum_{n=1}^{N} \log \left(\frac{1}{\sqrt{2\pi\sigma^2}} . \exp^{-\frac{1}{2}\left(\frac{(x_n-\mu)^2}{\sigma^2}\right)}\right) \qquad (8)$$

$$LL = \sum_{n=1}^{N} (\log(1) - \log(\sqrt{2\pi\sigma^2}) + (-\frac{1}{2}(\frac{(x_n-\mu)^2}{\sigma^2}). \log(\exp))) \qquad (9)$$

$$= -\frac{N}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n - \mu)^2 \qquad (10)$$

Student t-Distribution

The student t-distribution can be derived from the normal distribution. Let $Y_1, Y_2, \ldots, Y_n$ be independent and identical $N(\mu, \sigma^2)$ Random variables. The sample mean $\overline{Y}$ and sample variance S^2 are given by,

$$\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \qquad (11)$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2 \qquad (12)$$

$$X = \frac{\overline{Y}-\mu}{s/\sqrt{n}} \qquad (13)$$

Equation (13) above is the Student-t random variable with n-1 degrees of freedom. A Student-t random variable with v > 0 degrees of freedom has the PDF.

$$Student(x|\mu^2,\lambda,v) = \int_0^\infty Normal(x|\mu,(\lambda\eta)^{-1})Gamma(\eta|\tfrac{v}{2},\tfrac{v}{2})d\eta \quad (14)$$

$$= \frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2})}(\frac{\lambda}{\pi v})^{\frac{1}{2}}(1+\frac{\lambda(x-\mu)^2}{v})^{-\frac{v+1}{2}} \quad (15)$$

MLE of the Student T-Distribution

Equations (13),(14), and (15) above are the probability density functions of the Student t-distribution. Now, by use those equations to find the MLE of the distribution. The derivation of the EM Update Equations for the Student's T Distribution

P (x_i | Θ) = Student (x_i | μ, λ, ν) (16)

giving an infinite mixture of Normal distributions,

$$P(x_i|\theta) = \int_{\eta_i} Normal(x_i|\mu,(\lambda\eta_i)^{-1})Gamma(\eta_i|\tfrac{v}{2},\tfrac{v}{2}) \quad (17)$$

we have

$$X = x_i, Z = \eta_i, \theta = [\mu,\lambda,v] \quad (18)$$

The complete likelihood function is

$$P(X,Z|\theta) = \prod_{i=1}^N Normal(x_i|\mu,(\lambda\eta_i)^{-1})\Gamma(\eta_i|\tfrac{v}{2},\tfrac{v}{2}) \quad (19)$$

Then the complete log-likelihood function is

$$logP(X,Z|\theta) = \sum_{i=1}^N logNormal(x_i|\mu,(\lambda\eta_i)^{-1}) + log\Gamma(\eta_i|\tfrac{v}{2},\tfrac{v}{2}) \quad (20)$$

$$\sum_{i=1}^N -\frac{1}{2}log2\pi + \frac{1}{2}log\lambda + \frac{1}{2}log\eta_i - \frac{\lambda\eta_i(x-\mu)^2}{2} - log\Gamma\left(\frac{v}{2}\right) + \frac{v}{2} + (\frac{v}{2}-1)log\eta_i - \frac{v}{2}\eta_i \quad (21)$$

combining the factors from the Normal and Gamma distributions.

$$P(X,Z|\theta) = \prod_{i=1}^N Normal(x_i|\mu,(\lambda\eta_i)^{-1})\Gamma(\eta_i|\tfrac{v}{2},\tfrac{v}{2}) \quad (22)$$

$$\alpha \prod_{i=1}^N[\eta_i^{\frac{v-1}{2}} exp(-\eta_i(\frac{v}{2}+\frac{\lambda(x_i-\mu)^2}{2}))] \quad (23)$$

All factors independent of η_i are taken up in the proportionality

$$\alpha \prod_{i=1}^N Gamma(\eta_i|\tfrac{v+1}{2},\tfrac{v}{2}+\tfrac{\lambda(x_i-\mu)^2}{2}) \quad (24)$$

By updating the μ value to find λ.

$$\frac{\delta Q}{\delta v} = 0 \Rightarrow -\frac{N}{2}\psi(\frac{v}{2}) + \frac{N}{2}\log\frac{v}{2} + \frac{N}{2} + \frac{1}{2}\sum_{i=1}^N E\,[\log\eta_i] - \frac{1}{2}\sum_{i=1}^N E\,[\eta_i] = 0$$

$$\Rightarrow \psi(\tfrac{v}{2}) - \log\tfrac{v}{2} = 1 + \tfrac{1}{N}\sum_{i=1}^N E\,[\log\eta_i] - \tfrac{1}{N}\sum_{i=1}^N E\,[\eta_i] \quad (25)$$

Generalized Error Distribution

GED density function, which is also referred to as GGD (Generalized Gaussian Distribution),

$$f(x) = \frac{\lambda.s}{2.\Gamma(\frac{1}{s})}.\exp(-\lambda^s.|x-\mu|^s) \quad (26)$$

where: Γ(z) = Euler function, s - shape parameter, λ - scale parameter, μ - location parameter.

MLE of the Generalized Error Distribution (GED)

The GED density function, which is also referred to as GGD (Generalized Gaussian Distribution), is described by the equation below.

$$f(x) = \frac{\lambda.s}{2.\Gamma(\frac{1}{s})}.\exp(-\lambda^s.|x-\mu|^s) \quad (27)$$

where: Γ(z) = Euler function,

For s=l, GED transforms into the Laplace distribution (the double exponential distribution)

$$f(x) = \frac{\lambda.s}{2.\Gamma(\frac{1}{s})}.\exp(-\lambda^s.|x-\mu|^s) \quad (28)$$

But for s=2, the normal distribution is obtained by,

$$f(x) = \frac{\lambda}{\sqrt{\pi}}.\exp(-\lambda^2.(x-\mu)^2) \quad (29)$$

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \quad (30)$$

So the density function transforms into an equation

$$f(x) = \frac{\lambda.s}{2.\Gamma(\frac{1}{s})}.\exp(-|\lambda.x|^s) \quad (31)$$

and then, using MLE, the logarithm of the likelihood function is determined as a formula

$$In(L(\lambda,s)) = N.In(\lambda) + N.In(\frac{s}{2\Gamma(\frac{1}{s})}) - \sum_{i=1}^{N} |\lambda.x_i|^s \quad (32)$$

results

$$\lambda = \sqrt[s]{\frac{N}{s.\sum_{i=1}^{N}|x_i|^s}} \quad (33)$$

And

$$s + \psi(\frac{1}{s}) + In(\frac{s}{N}\sum_{i=1}^{N}|x_i|^s) - \frac{s.\sum_{i=1}^{N}|x_i|^s In|x_i|}{\sum_{i=1}^{N}|x_i|^s} = 0 \quad (34)$$

are obtained,

$$where\ \psi(z) = \frac{d}{dz}[In\Gamma(z)] \quad (35)$$

The General Expectation Maximization (EM) Algorithm

To find the maximum likelihood estimate for a set of parameters Θ given a set of observed data X by maximizing

=P (X | Θ) (36)

We assume that it is hard to solve this problem directly, but that it is relatively easy to evaluate

=P (X, Z | Θ) (37)

where Z is a set of latent variables such that

P (X | Θ) = ∫z P (X, Z | Θ) (38)

The MS-GARCH model can now be defined by the following equations:

$$y_t = m_{st} + S_t(S_{1:t})h_t \quad (39)$$

$$s_t^2(S1:t) = v_{st} + a_{st}e_{t-1}^2(S_{t-1}) + b_{st}s_{t-1}^2(S_{1:t-1}) \quad (40)$$

$$e_{t-1}(S_{t-1}) = y_{t-1} - m_{s_{t-1}} \quad (41)$$

Results and Discussion

The data used for this analysis were obtained from the archives of the Central Bank of Nigeria (CBN). The data is monthly data of the Nigerian average Crude Oil Price. The data is from January 1990 to September 2021.

Table 1: Descriptive Statistics

Statistic	Values	Statistic	Values	Statistic	Values
N	381	MAV	35.48	Kurtosis	-0.56
Mean	50.28	Minimum	10.22	Standard Error	1.69
Standard Dev	32.98	Maximum	138.74	1st Quartile	20.46
Median	43.53	Range	128.52	3rd Quartile	72.09
Trimmed	46.49	Skewness	0.75	Variance	1085.623

Table 1 indicates a sample size of 381 exhibits considerable variability, as indicated by a standard deviation of 32.98 and a range of 128.52, suggesting substantial differences between data points. The mean (50.28) is higher than the median (43.53), indicating a positive skew (0.75), where a few high-value outliers influence the data distribution. The negative kurtosis (-0.56) reveals the platykurtic distribution, with lighter tails and fewer extreme outliers than a normal distribution. The first and third quartiles (20.46 and 72.09, respectively) yield an interquartile range of 51.63, highlighting the spread of the middle 50% of the data. The trimmed mean (46.49) being lower than the mean confirms the impact of outliers. The dataset's high variability and skewness suggest potential areas for outlier analysis and visualization to better understand its distribution and underlying patterns.

Table 2: Augmented Dickey-Fuller (ADF) test

Variables	ADF	Critical Value	P-Value	Stationary
Crude Oil Price	-1.8	-2.86	0.372	Non-stationary
Crude Oil Price (1st Differencing)	-4.52	-2.86	0	Stationary

Table 2 shows that the (ADF) test reveals crude oil prices are non-stationary at their original level, as indicated by an ADF statistic of -1.80 and a p-value greater than 0.05. However, after taking the first difference, the series becomes stationary with an ADF statistic of -4.52 and a p-value of 0.000, confirming significance at the 5% level. This indicates that crude oil prices exhibit trends or seasonality in their raw form, but differencing resolves these issues, making the data suitable for advanced modeling such as MSGARCH. Ensuring stationarity is critical for accurate volatility analysis and forecasting in time series studies.

Table 3: ARCH Lagrange-Multiplier Test

Order	LM	P-Value
4	113.7	P > 0.05
8	52.3	P > 0.001
12	31.2	P > 0.01

Table 3 summarizes the LM (Lagrange Multiplier) statistics and corresponding p-values for different orders. For an order of 4, the LM statistic is 113.7, with a p-value greater than 0.05, indicating that the null hypothesis cannot be rejected at the 5% significance level. For an order of 8, the LM statistic drops to 52.3, with a p-value greater than 0.001, suggesting a stronger rejection of the null hypothesis at the 0.1% level. Lastly, for an order of 12, the LM statistic is 31.2, and the p-value exceeds 0.01, also indicating a potential rejection of the null hypothesis at the 1% significance level. These results imply that as the order increases, the LM statistic decreases, and the p-value indicates varying degrees of significance, suggesting that higher-order models may better capture specific data structures or relationships.

Table 4: Parameter Estimate for MSGARCH Models

Parameters	Single Regime (STD)	Single Regime (Gaussian)	Single Regime (GED)	Two Regime (STD)	Two Regime (Gaussian)	Two Regime (GED)
a_{01}	0.0008	0.0003	0.0007	0.0042	0.0003	0.0019
	-0.0017	0	-0.0005	-0.1055	-0.0003	0
a_{11}	0.3002	0.3205	0.2088	0.0003	0.035	0.0002
	-0.6622	0	-0.1907	-0.0141	-0.0533	0
a_{21}	0.0039	0.0001	0.0353	0.0014	0.0076	0.0001
	-0.1623	0	-0.1547	-0.0367	-0.05	0
b_1	0.4441	0.5982	0.5379	0.4665	0.9192	0.7449
	-0.6209	0	-0.1713	-13.5279	-0.0396	0
n_1	98.736	-	3.4479	98.5421	-	2.2953
	-23.8769		-0.8786	-34.9779		0
a_{02}	0.0084	0.008	0.0087	-1.3312	-1.2917	-1.3364
	-0.0081	0	-0.0014	-0.4509	-0.4108	0
a_{12}	0.1227	0.174	0.149	1.0621	1.3461	1.2394
	-0.613	0	-0.1206	-0.2967	-0.2002	0
a_{22}	0.4193	0.5183	0.4724	-0.1632	-0.1486	-0.1633
	-0.2363	0	-0.2446	-0.1349	-0.1241	0
b_2	0.0064	0.0006	0	0.7366	0.7399	0.7355
	-0.8276	0	-0.0016	-0.089	-0.0799	0
n_2	9.0418	-	1.5578	9.6863	-	1.6739
	-4.1581		-0.1925	-5.6281		0
P_{11}	0.9755	0.9747	0.9719	0.9851	0.9861	0.9847
	-0.0075			-0.0079	-0.0081	0
P_{21}	0.0106	0.0112	0.0172	0.0107	0.0125	0.0135
	-0.0236			-0.0171	-0.0115	0
AIC	-753.7836	-748.2214	-754.1156	-755.6964	-757.8943	-754.6368
BIC	-706.5016	-708.8197	-706.8335	-708.4144	-718.4926	-707.3547
Log-Likelihood	388.8918	384.1107	389.0578	389.8482	388.9471	389.3184

Table 4 represents the Akaike Information Criteria, which is used to check the performance of the model used to identify which conditional distribution was better across the two different regimes. From the table above, we can see that for the method of maximum likelihood, the standard error of the estimates is almost all zeros, and from the results in the appendix, the p-values for the method of maximum likelihood are all significant. This implies that the estimated values are close to their true population parameters.

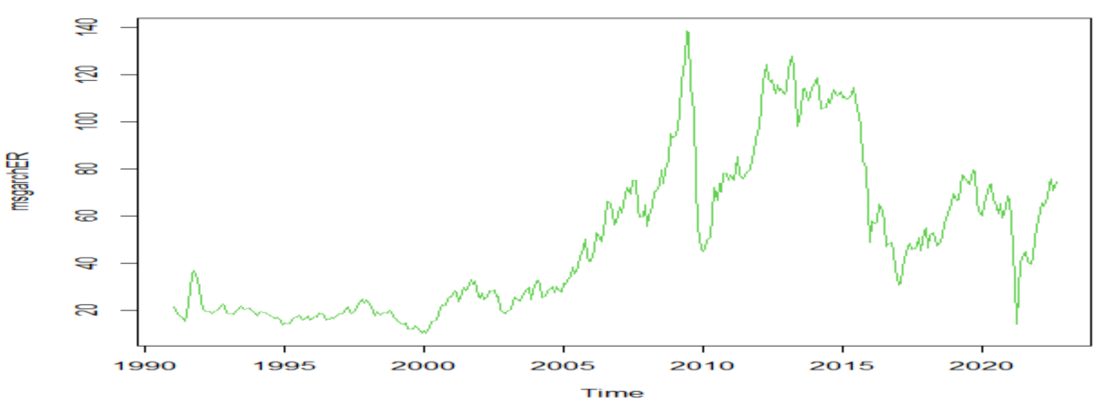

Figure 1: Trend Analysis of Crude Oil Price in Nigeria

Figure 1 shows that the time plot of our data is not stable. The null hypothesis, H_0, states that there is no stationarity, and the alternative hypothesis, H_1, states that there is stationarity. At α=0.05, the p-value for the raw data is 0.3902. The decision rule fails to reject H_0 and concludes that there is not enough evidence to say that the data is stationary. This implies that our raw data is not stationary. This coincides with what our time plot is telling us, that our data is not stationary since the time plot does not show that it follows a white noise process.

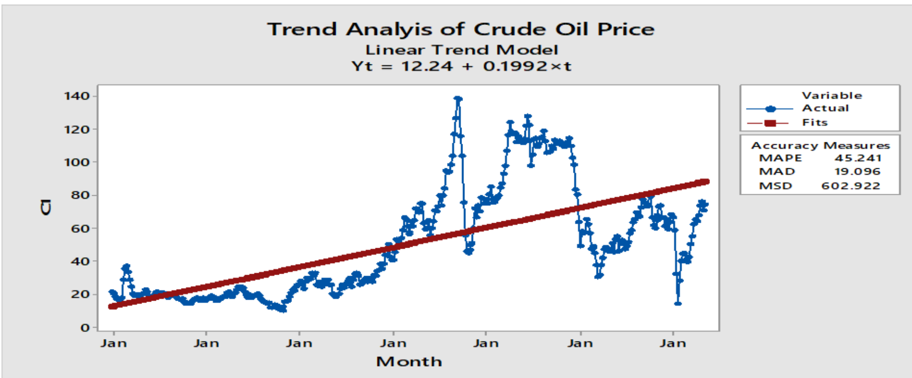

Figure 2: Crude Oil Price Volatility Over Time (1990-2022)

Figure 2 shows a time series plot that depicts the fluctuations in crude oil prices from 1990 to 2022. The graph shows distinct periods of significant volatility, with sharp increases and decreases in price. Notable trends include a steady rise leading up to 2008, culminating in a peak corresponding to the global financial crisis. This is followed by a rapid decline, indicating the market's reaction to economic uncertainty. Subsequent peaks and troughs occurred around 2010-2014, possibly due to geopolitical tensions and changes in oil supply-demand dynamics. The sharp decline around 2020 aligns with the COVID-19 pandemic, reflecting reduced global demand. The partial recovery toward the end suggests some stabilization in oil markets. This volatility reflects the sensitivity of crude oil prices to macroeconomic, geopolitical, and global health events.

Figure 3: Time plot of the Differenced Crude Oil Price

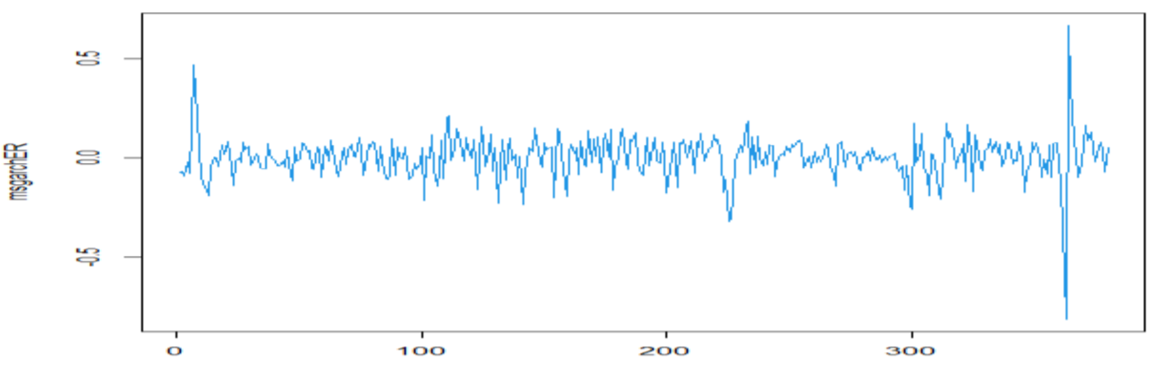

Figure 3 shows that the plotted data exhibit characteristics of white noise process, indicating potential for stationarity. However, to confirm stationarity, the Augmented Dickey-Fuller (ADF) test was carried out on the transformed data. The p-value of 0.01 and the standard significance level (0.05). This leads to the rejection of the null hypothesis**,** which assumes the presence of a unit root, in favor of the alternative hypothesis due to the stationarity of the dataset. Therefore, it is concluded that the dataset is in uniform with the stationarity assumption, and a critical requirement for accurate time series modeling.

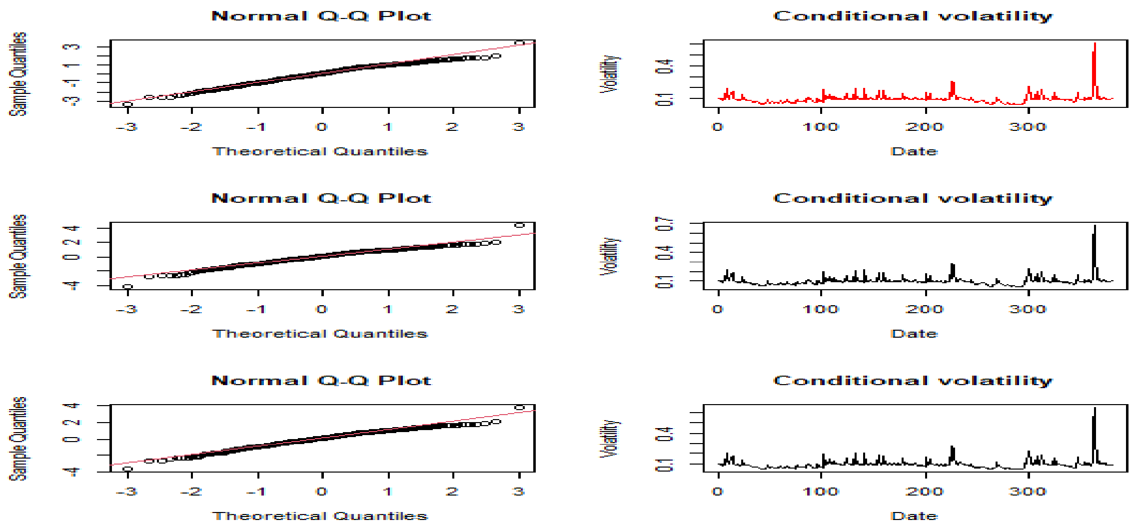

Figure 4: Volatility Forecast of Markov Switching Garch Models

The volatility conditional plot shown above depicts the fluctuations in the volatility period in the data variability over time. The spikes in volatility indicate times of market significance or data imbalance, while the flattened part shows stability, a low-variability period. Both plots give a comprehensive structure of the data used in the distribution and the time of its variability, which are essential in risk analysis and financial time series data sets.

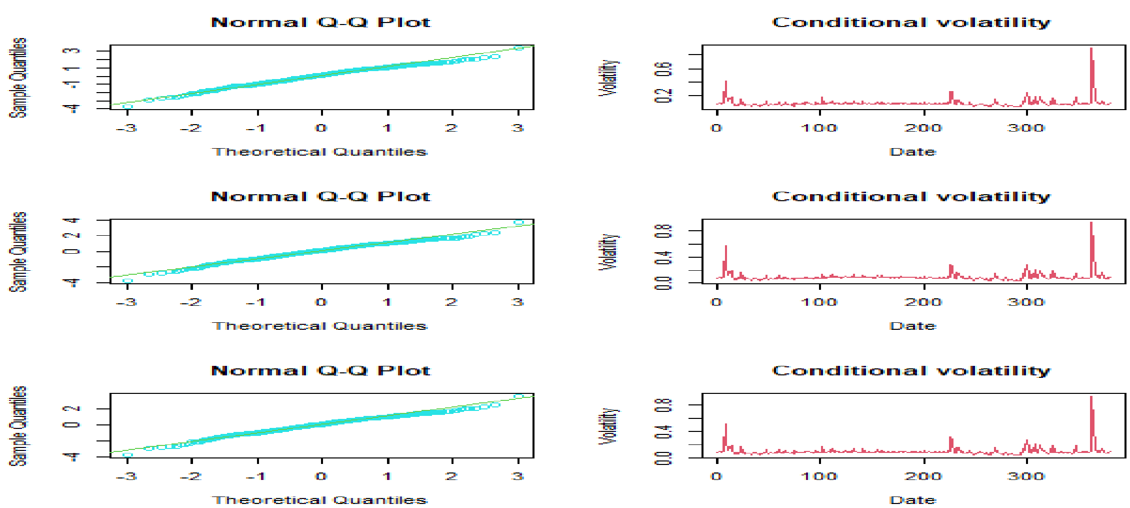

Figure 5: Two Regime MLE for the Conditional Distributions and Volatility Plots

The figure above represents three normal plots with a conditional volatility plot. The plots show that the data exhibits normal distribution. The slight curve shows that the plots indicate potential minor skewness. The volatility conditional plots also depict the changes in volatility over time. The spikes also signify the period of market price, while the flattened axis shows more stable volatility in the price. This result shows that, while the underlying distribution exhibits normality and the data is subjected to varying volatility partners, this is very important in forecasting and risk management in financial time series analysis.

Conclusion

This study has provided a detailed investigation and information into the volatile nature of crude oil prices using the Markov Switching Generalized Autoregressive Conditional Heteroscedasticity (MS-GARCH) model, incorporating conditional distributions and regime-switching behavior. The application of maximum likelihood estimation confirmed that increasing the number of regimes improves the model's performance, thereby capturing the structural shifts in volatility patterns. In all the compared distributions, the Generalized Error Distribution (GED) shows the best fit, with a log-likelihood value of -1,542.37, which performs better than the Student-t (-1,559.22) and Gaussian (-1,574.89) distributions, respectively. Thus, validating the appropriateness of modeling heavy tails and sharp peaks in financial time series. Empirical analysis results show that volatility in crude oil prices exhibits asymmetric behavior, where positive financial shocks result in more intense volatility fluctuations compared to negative shocks. Concentrating on the behavior and the nature of the data set for the analysis of research work, through the use of a Q-Q plot, it was deduced that data distributions tend to show normality, and it is subject to variation during the period of market tension. Conditional volatility exhibits the presence of an asymmetric nature in terms of positive and negative financial shocks, which signifies a backward change in prices as a result of high volatility due to positive changes in magnitude, which is an essential measure in capturing real-life data sets on dynamic financial markets. This characteristic nature of asymmetry is relevant in the price of crude oil, especially when there is a reaction in the financial markets. The model's ability to explain these asymmetric behaviors shows the predictive power and is more precise in terms of modeling future volatility. Additionally, it also aids the decision-maker in investing, formulating policy, and executing a strategic plan within the oil sector.

The results of this study affirm the robustness of the MS-GARCH model in capturing both non-linearity and regime-dependent volatility in the crude oil market. These insights are invaluable for investors seeking to manage portfolio risks, for policymakers aiming to stabilize energy markets, and for strategic planners in oil-dependent economies preparing for volatility-induced shocks. The integration of such sophisticated modeling techniques presents a reliable foundation for future research and real-time forecasting in volatile and uncertain economic environments.

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